Remarks on the Origins of Path Integration: Einstein and Feynman*

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Abstract

I offer some historical comments about the origins of Feynman's path integral approach, as an alternative approach to standard quantum mechanics. Looking at the interaction between Einstein and Feynman, which was mediated by Feynman's thesis supervisor John Wheeler, it is argued that, contrary to what one might expect, the significance of the interaction between Einstein and Feynman pertained to a critique of classical field theory, rather than to a direct critique of quantum mechanics itself. Nevertheless, the critical perspective on classical field theory became a motivation and point of departure for Feynman's space-time approach to non-relativistic quantum mechanics.

Keywords: History of quantum mechanics, Einstein, Feynman.

1 Introduction

In this paper, I am interested in the genesis of Feynman's path integral approach to non-relativistic quantum mechanics. I take Feynman's 1948 paper on "A Space-Time Approach to Quantum Mechanics"[1] as the point in time when the approach was fully formulated and published and made available to the community of physicists. I will take a look into the prehistory of Feynman's 1948 paper. I shall not attempt to give anything like a balanced, or even complete historical account of this prehistory. Instead, I will focus on a little footnote in Feynman's paper:

> The theory of electromagnetism described by J.A. Wheeler and R.P. Feynman, Rev. Mod. Phys. **17**, 157 (1945) can be expressed in a principle of least action involving the coordinates of particles alone.

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> It was an attempt to quantize this theory, without reference to the fields, which led the author to study the formulation of quantum mechanics given here. The extension of the ideas to cover the case of more general action functions was developed in his Ph.D. thesis, "The principle of least action in quantum mechanics" submitted to Princeton University, 1942. [1, p. 385]

My guide in organizing my remarks will be to look at what we know about any direct and indirect interaction between Feynman and Einstein. Let me briefly motivate this focus on Einstein and Feynman.

Feynman was born in New York in 1918, did his undergraduate studies at MIT, and took his Ph.D. with John A. Wheeler at Princeton University in 1942, before going to Los Alamos during the war years. After the war, he was first at Cornell and in 1951 he went to Caltech. In 1954, Feynman received the Einstein Award,[2] as a 36-year old man for his work on quantum electrodynamics that in 1965 would earn him the Nobel prize for physics.

The Einstein Award was a prestigious award, established in 1949 in Einstein's honor, but it seems that Einstein had not much to do with the awarding of the prize to Feynman.

At the time of Feynman's receiving the Einstein award, Einstein himself was a 76-year old world famous man. He had been living in Princeton since his emigration from Nazi-Germany in 1933 and was scientifically engaged in a search for a unified field theory of gravitation and electromagnetism.[3] But he also still thought about problems of the foundations of quantum mechanics. Among his extensive research notes and manuscripts with calculations along the unfied field theory program, there is, e.g., a manuscript page from around 1954 with a concise formulation of Einstein's of the famous Einstein-Podolsky-Rosen incompleteness argument for standard quantum mechanics. Probably in reaction to David Bohm's reformulation of the original argument, Einstein here also formulates the incompleteness argument for spin observables.[4]

With both Feynman and Einstein being concerned with the foundations of quantum mechanics, one might hope that an interaction between the two physicists, if there was any, might give us some insight into the historical development of our understanding of the principles of quantum theory.

A similar question was also asked once by Wheeler. In 1989, after Feynman's death, he recalled:

> Visiting Einstein one day, I could not resist telling him about Feynman's new way to express quantum theory.[5]

After explaining the basic ideas of Feynman's path integral approach to Einstein, Wheeler recalls to have asked:

> "Doesn't this marvelous discovery make you willing to accept quantum theory, Professor Einstein?" He replied in a serious voice, "I still cannot believe that God plays dice. But maybe," he smiled, "I have earned the right to make my mistakes."[5]

So is this the end of my story?

According to Feynman's own account, he himself met Einstein only twice. One of these encounters was at the occasion of Feynman's first technical talk, as a young graduate student, in the Princeton physics department. The occasion probably took place in late 1940. Wheeler had suggested that Feynman was to talk on their joint work, and Feynman recalls

> Professor Wigner was in charge of the colloquium, so after I said I would do it, he told me that he had heard from Wheeler about the work and he knew something about it. I think we had discussed it a little bit with him. And he thought it was important enough that he had taken the liberty to invite especially Professor Henry Norris Russell from the astronomy department, the great astronomer, you know, John von Neumann from the mathematics department, the world's great mathematician, and Professor Pauli, who was visiting from Zurich, would be there. And Professor Einstein had been especially invited—and although he never comes to the colloquia, he thinks he will come!
>
> So I went through fire on my first. I must have turned a yellowish-green or something [...].[1]

Feynman continues to recount details of this seminar, he relates how his excitement and anxiety abated once he started to talk about physics, and indicates how some members of his audience, including Einstein, reacted to his presentation in question time.

In the following, I will take this encounter between Feynman and Einstein as a point of departure for a historical argument: the interaction between Feynman and Einstein reminds us of a significant historical context of discovery of the path integral method. This original context is still prominently visible in Feynman's 1942 thesis[10] but it is already reduced to a footnote in his 1948 publication.[1] For an appreciation of the path-integral method, even today, it may nevertheless still be useful to recall the historical circumstances of its discovery.

Specifically, I will address and discuss the following four questions:

1. What is the Wheeler-Feynman theory that Feynman presented in his first seminar at Princeton?

2. What does Einstein have to do with this?

3. What does this have to do with path integrals?

4. Why is this context of the origin of the path integral approach only mentioned in a footnote in Feynman's 1948 paper?

[1] [7, p. 133]. I am quoting from the transcript of an oral history interview conducted by C. Weiner with Feynman in 1966. For slightly different versions of the episode, see also [8, pp. 64f] and [9, p. 66].

Most of the information that the argument is based on can be found in Schweber's book on the history of quantum electrodynamics.[11] The significance of the Wheeler-Feynman theory for Feynman's subsequent development is also emphasized by Feynman himself in his Nobel lecture.[12]

2 The Wheeler-Feynman absorber theory

The results of their joint work that Feynman presented in the Princeton physics colloquium were not published at the time. Feynman gave his presentation again, shortly thereafter, at a meeting of the American Physical Society in Cambridge, Massachussetts, that took place on 21 and 22 February 1941. Of this talk, an abstract was published.[13] There exists also a typescript by Feynman giving an account of the theory,[14] dated to spring 1941 [11, p. 383]. The abstract identifies radiative damping as a problem in Lorentz's classical electron theory and in Dirac's theory of a point electron, and then summarizes the main points of Feynman's paper:

> We postulate (1) that an accelerated point charge in otherwise free space does *not* radiate energy; (2) that, in general, the fields which act on a given particle arise only from *other* particles; (3) that these fields are represented by one-half the retarded plus one-half the advanced Liénard-Wiechert solutions of Maxwell's equations. In a universe in which all light is eventually absorbed, the absorbing material scatters back to an accelerated charge a field, part of which is found to be independent of the properties of the material. This part is equivalent to one-half the retarded field *minus* one-half the advanced field generated by the charge. It produces radiative damping (Dirac's expression) and combines with the field of the source to give retarded effects alone.[13]

A detailed account of the Wheeler-Feynman theory was published after the war in two papers. The first paper appeared in 1945 under the title 'Interaction with the Absorber as the Mechanism of Radiation.'[15] As indicated in a first footnote to the title, this paper essentially gives an account of the theory that Feynman had presented in 1941.

In a long second footnote to the title of the paper, Wheeler then explains that this first paper was actually planned to be the third part of a projected series of five papers. Of the missing four parts, only the second one actually appeared, four years later, in 1949, under the title 'Classical Electrodynamics in Terms of Direct Interparticle Action'.[16]

The core of the Wheeler-Feynman theory thus concerned a special problem that arose out of a broader research program, laid out, in part, in the later 1949 paper. In qualitative terms, the broader research program concerned this.

Among the many difficulties with attempts to come to a quantum theory of electrodynamics in the late thirties, Wheeler and Feynman thought some had to do with difficulties that occur already at the level of classical electrodynamic

field theory. As a radical response, Wheeler and Feynman questioned whether the notion of an electromagnetic field is, in fact, a useful one. They argued that one should in principle be able to express all electromagnetic phenomena in terms of direct interaction between point-like particles. Any notion of a field would be a derived concept. The primary notion would be a collection of point-like charges that interact with each other through Liénard-Wiechert retarded and advanced potentials.

They found that such a theory is expressible in terms of an action principle that involves a variation over the world-lines of charged electrons. They wrote the action principle as[16]

$$\begin{aligned} J = - \quad & \textstyle\sum_a m_a c \int (-da_\mu da^\mu)^{\frac{1}{2}} + \sum_{a<b} (e_a e_b / c) \\ & \times \textstyle\iint \delta(ab_\mu ab^\mu)(da_\nu db^\nu) = \text{extremum}, \end{aligned} \tag{1}$$

where the sums are over electrons of mass m_a and charge e_a, da denotes derivative with respect to the respective proper time, and $ab^\mu \equiv a^\mu - b^\mu$ is short for the four-vector of the separation between the particles, a somewhat unusual notation introduced in order to be able to make use of the Einstein summation convention. The attractive feature of this action is that "all of mechanics and electrodynamics is contained in this single variational principle." [16, p. 425] Note that the single action principle incorporates both the Maxwell equations and the Lorentz force law. The idea and the action (1) were known before, they can be found, in more or less explicit terms, in older papers by Schwarzschild,[17] Tetrode,[18] and Fokker.[19]

The only problem with this formulation was the issue of radiative reaction. In classical theory, an accelerated electron radiates and loses energy to the field. To avoid the notion of a field, Wheeler and Feynman postulated that a single electron alone in the universe, if accelerated, would, in fact, *not* radiate. Instead, they succeeded to show that radiative reaction can arise in a universe with a surrounding material that absorbs all outgoing radiation. The electrons of the absorber interact with the electron at the source through advanced potentials, such that an accelerated electron feels a radiative force. This is the main point that Feynman was elaborating on in his Princeton seminar.

3 Einstein and the electromagnetic arrow of time

Feynman recalled that immediately after his presentation, Pauli asked critical questions and then asked Einstein whether he would agree.

> Anyway, Professor Pauli got up immediately after the lecture. He was sitting next to Einstein. And he says, "I do not think this theory can be right because of this, that and the other thing—" it's too bad that I cannot remember what, because the theory is not right, and the gentleman may well have hit the nail on the bazeeto, but I don't know, unfortunately, what he said. I guess I was too nervous to listen, and didn't understand the objections. "Don't you agree,

> Professor Einstein?" Pauli said at the end of his criticism. "I don't believe this is right—don't you agree, Professor Einstein?"
> Einstein said, "No," in a soft German voice that sounded very pleasant to me, and said that he felt that the one idea, the one thing that seemed to him, was that the principles of action and distance which were involved here were inconsistent with the field views, the theory of gravitation, of general relativity. But after all general relativity is not so well established as electrodynamics, and with this prospect I would not use that as an argument against you, because maybe we can develop a different way of doing gravitational interaction, too. Very nice. Very interesting. I remember that.[2]

We also know, both from Feynman [7, p. 133] and Wheeler [6, p. 167] as well as, independently, from a letter by Wheeler to Einstein, that Feynman and Wheeler visited Einstein once in his house in Princeton and discussed the "interpretation of the force of radiation in terms of advanced and retarded action at a distance."[20] It is unclear when the meeting took place,[3] and I am not aware of any detailed account of the discussion that took place, but it seems that Einstein alerted Feynman and Wheeler to existing literature on the subject, including some in which he himself was involved. In a footnote to their 1945 paper, Wheeler and Feynman acknowledge Einstein's input:

> We are indebted to Professor Einstein for bringing to our attention the ideas of Tetrode and also of Ritz, [...]. [15, n. 10]

Somewhere else in the article, they

> recall an inconclusive but illuminating discussion carried on by Ritz and Einstein in 1909, in which "Ritz treats the limitation to retarded potentials as one of the foundations of the second law of thermodynamics while Einstein believes that the irreversibility of radiation depends exclusively on considerations of probability." [15, p. 160]

The Einstein-Ritz controversy,[22–24] from which they quoted, was about the origin of irreversibility of electromagnetic radiation phenomena.[26] In the 1941 typescript, Feynman observed that their theory is in full agreement with Einstein's position against Ritz, that the fundamental electrodynamical equations are time-reversal invariant, and that the radiative irreversibility is a macroscopic, statistical phenomenon:

> The apparent irreversibility in a closed system, then, either from our point of view or the point of view of Lorentz is a purely macroscopic irreversibility. The present authors believe that all physical

[2] [7, p. 134], see also [8, p. 66], [9, pp. 67–68].

[3] It is even unclear whether the meeting in Einstein's house took place before or after Feynman's Princeton colloquium. In 1966, Feynman did not remember but was "pretty sure" that it was before the colloquium "because he knew me," see [7, pp. 133,139]. Wheeler [6, p. 167] recalls that it was "while working on our second action-at-a-distance paper", but from his letter to Einstein, we know that it must have been before November 1943, see also [21, p. 118].

phenomena are microscopically reversible, and that, therefore, all apparently irreversible phenomena are solely macroscopically irreversible. ([14, p. 13.1]; quoted in [11, p. 386].)

Feynman here has a footnote saying

> That this and the following statement are true in the Lorentz theory was emphasized by Einstein in a discussion with Ritz. (Einstein and Ritz, Phys. Zeits. 10, p323, (1909)). Our viewpoint on the matter discussed is essentially that of Einstein. (We should like to thank Prof. W. Pauli for calling our attention to this discussion.) (ibid.)

Although Pauli is credited here for alerting Feynman to the Ritz-Einstein controversy, we may assume that the point was also a topic when Feynman and Wheeler discussed their ideas with Einstein during their visit at his Princeton home. There is, in any case, an English translation, in Feynman's hand, of the Ritz-Einstein controversy[24] in the Feynman papers.[25]

4 Path integrals for actions with no Hamiltonian

In 1942, Feynman was recruited for the Los Alamos project. Before leaving for Los Alamos, Wheeler urged Feynman to write up his thesis.[27] Feynman's thesis[10] is not directly dealing with the Wheeler-Feynman absorber theory but it rather gives a discussion of the 'Principle of Least Action in Quantum Mechanics', and is, in fact, a direct forerunner of Feynman's 1948 paper. But the thesis is very explicit about its original motivation. The discussion of quantizing systems expressed in terms of a Lagrangian is given in the context of solving the general problem of finding a quantum version of the Wheeler-Feynman theory of action-at-a-distance. The main point here is that

> the theory of action at a distance finds its most natural expression in a principle of least action, which is of such a nature that no Hamiltonian may be derived from it. That is to say the equations of motion of the particles cannot be put into Hamiltonian form in a simple way. This is essentially because the motion of one particle at one time depends on what another particle is doing at some other time, since the interactions are not instantaneous.[28]

This is not just a remark made in (a draft version of) the preface to motivate the approach. An example that derives directly from the action (1) is discussed also in the body of the text. At some point, Feynman explains how to generalize the quantization procedure to more general actions, for example those involving time-displaced interactions:

> The obvious suggestion is, then, to replace this exponent by $\frac{i}{\hbar}$ times the more general action. The action must of course first be expressed

> in an approximate way in terms of q_i, t_i in such a way that as the subdivision becomes finer and finer it more nearly approaches the action expressed as a functional of $q(t)$.
> In order to get a clearer idea of what this will lead to, let us choose a simple action function to keep in mind, for which no Hamiltonian exists. We may take,
>
> $$\mathcal{A} = \int_{-\infty}^{\infty} \left\{ \frac{m\dot{x}(t)^2}{2} - V(x(t)) + k^2 \dot{x}(t)\dot{x}(t+\tau) \right\} dt,$$
>
> which is an approximate action function for a particle in a potential $V(x)$ and which also interacts with itself in a mirror by half advanced and half retarded waves, [...]. [10, p. 41]

In the 1941 typescript Feynman comes close to showing how this simple action follows from the general action (1) by considering the special case of two charges at a distance apart in otherwise free space, neglecting their electrostatic interaction. Of course, the path integral quantization of actions that are non-local in time is considerably more involved[29] and Feynman does not give an explicit discussion of his example. Nevertheless, it confirms his remark in the (actual) preface of the thesis which

> is concerned with the problem of finding a quantum mechanical description applicable to systems which in their classical analogue are expressible by a principle of least action, and not necessarily by Hamiltonian equations of motion. [10, p. 6]

5 The demise of the early context of path integration

In 1949, even before the second of the Wheeler-Feynman papers appeared in print, Feynman himself submitted another one of his famous papers, entitled 'Space-Time Approach to Quantum Electrodynamics.'[30] In it, one finds this little footnote:

> These considerations make it appear unlikely that the contention of J.A. Wheeler and R.P. Feynman, Rev. Mod. Phys. **17**, 157 (1945), that electrons do not act on themselves, will be a successful concept in quantum electrodynamics. [30, p. 773]

Why did Feynman retract a basic assumption of his joint work with Wheeler, with explicit reference to their earlier paper? Two years later, Feynman wrote a letter to Wheeler asking him about his opinion about the status of their earlier work:

> I wanted to know what your opinion was about our old theory of action at a distance. It was based on two assumptions:

> (1) Electrons act only on other electrons;
> (2) They do so with the mean of retarded and advanced potentials.
> The second proposition may be correct but I wish to deny the correctness of the first. The evidence is two-fold. First there is the Lamb shift in hydrogen which is supposedly due to the self-action of the electron. [...]
> The second argument involves the idea that the positrons are electrons going backwards in time. [...]
> So I think we guessed wrong in 1941. Do you agree?[31]

I am not aware of an explicit response by Wheeler to this letter, but several remarks in his autobiography[6] indicate that he, too, eventually gave up his belief in an action-at-a-distance electrodynamics: "[...] until the early 1950s, I was in the grip of the idea that Everything is Particles." [6, p. 63]

For Feynman, one of the two reasons for giving up the theory of action-at-a-distance was an experimental finding, the Lamb shift. Lamb had presented data from his experiments on the fine structure of hydrogen at the Shelter Island conference. This conference, devoted to problems of the quantum mechanics of the electron, took place in June 1947 and was an event of considerable impact in the history of post-war physics [11, ch. 4]. It brought together the leading theorists for the first time after the war for a meeting which helped to determine the course of American physics in the atomic age. 9 of the 23 participants ended up being awarded the Nobel prize, a significant fraction of the participants were of the young generation. It was at this conference that Feynman presented his 'space-time approach to quantum mechanics', essentially the work of his thesis, and soon after the conference he penned his classic 1948 paper.[1]

Incidentally, the Shelter Island conference could have provided an occasion for a third encounter between Feynman and Einstein: following a suggestion of Wheeler, who was present as well, Einstein was among the invitees but he declined, due to ill health [11, pp. 169f]. It is tempting to speculate how Einstein would have reacted to Feynman's presentation of his new approach to quantum mechanics at this meeting.

When Feynman wrote up his approach for publication, he decided to mention the original motivation for his work only in passing. Given the generality of the path integral formulation, it may be seen a wise decision on Feynman's part to reduce the historical context of its genesis to a footnote.

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